

The Wilmott CQF course provides lots of additional lectures to its alumni.
These are called CQF Extras. See <http://www.cqf.com/extras> for details.
I gave one such talk, based on this presentation.

CQF 'Extras' Lecture

Commodities Modelling



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Wilmott CQF, 2008

MSc Finance and Commodities, Birkbeck, Univ of London, 2008

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wsmith © ems.bbk.ac.uk

will © commoditymodels.com

What we will cover:



What Are Commodities?



Energy Modelling :
Oil, Gas, Electricity



Commodity Correlation

What is a Commodity?



A distinct asset class...

Some common properties, but...

Every commodity is unique.



What is a Commodity?

Something
physical, widely
used

Energy
Products

Metals

Agriculturals

Exotics

Standardised
product

Quantity

Quality

Time

Place



What are the major commodities?

Energy

- oil, natural gas, electricity, heating oil, gasoline, ...

Metals

- gold, copper, iron, ...

Agriculturals

- wheat, corn, soya, ...

Exotics

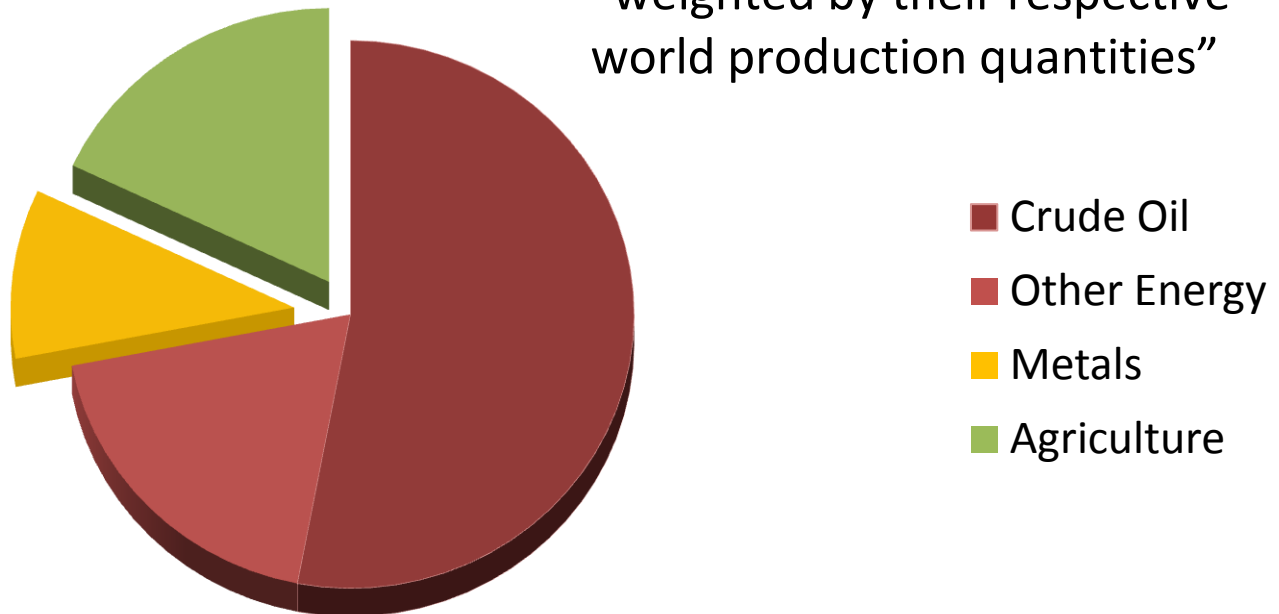
- shipping, CO2 emissions, weather derivatives



What are the major commodities?

Goldman Sachs Commodity Index (GSCI) Composition (Oct 2009)

“weighted by their respective world production quantities”

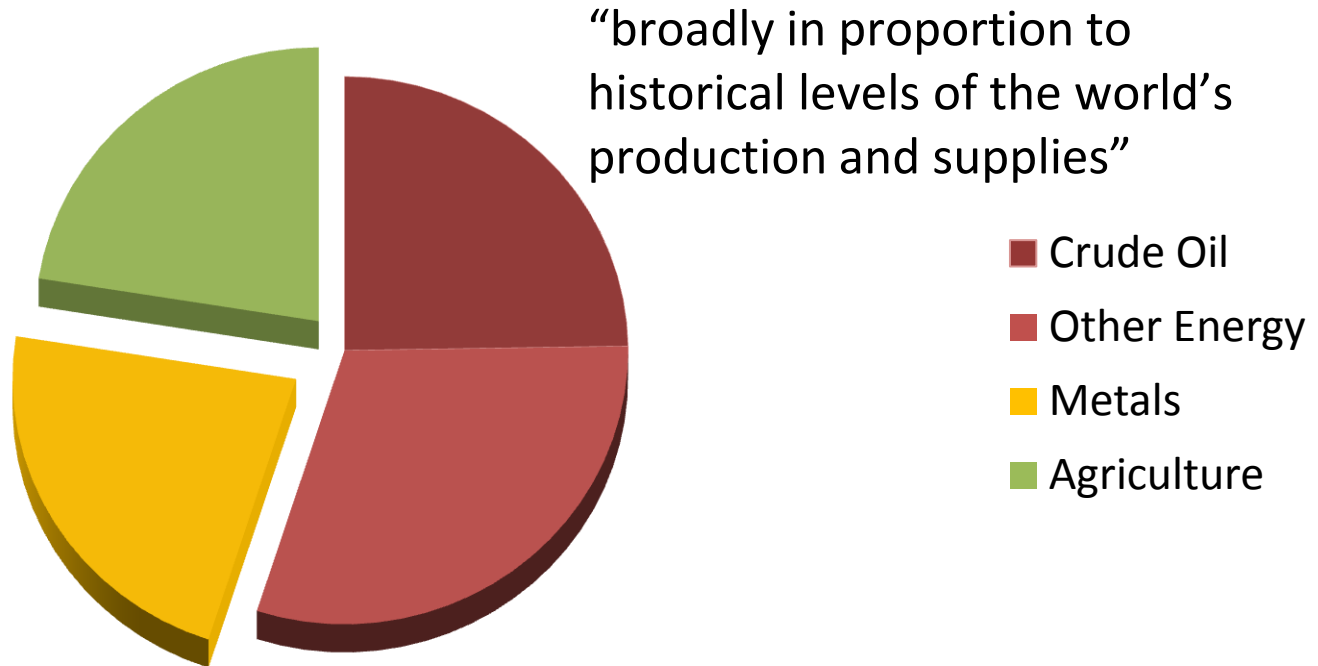


From <http://www2.goldmansachs.com/services/securities/products/sp-gsci-commodity-index/tables.html>

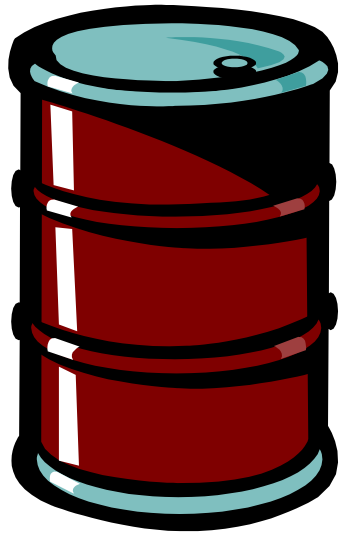


What are the major commodities?

Deutsche Bank Commodity Index (DBC) Composition (Oct 2009)



From http://dbfunds.db.com/Dbc/Pdfs/DBC_Fact_Sheet.pdf



Crude Oil

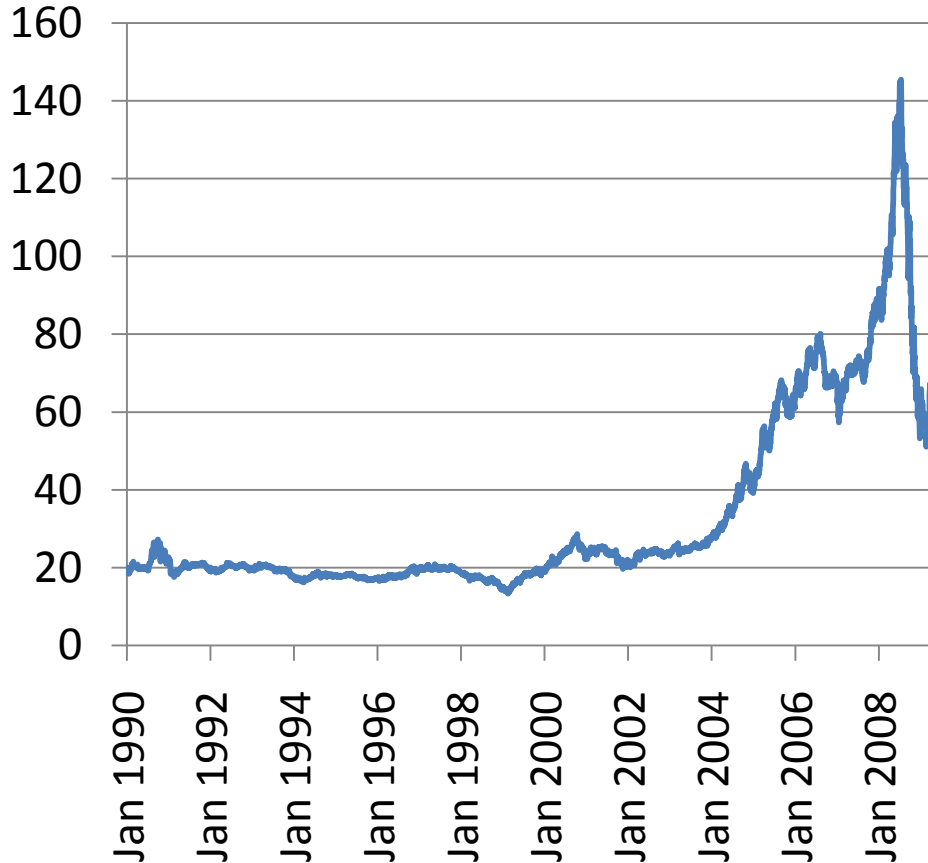
Biggest Energy Market
Simplest to Model*

* Only spot price.
And even then I'm probably lying...



Crude Oil - Price

WTI Spot Price



Mean Reverting?

Random Walk?

What is WTI anyway?



Crude Oil – Basic Spot Model

Mean Reverting?
(E.g. O-U)

$$dS_t = a(b - S)dt + \sigma dW_t$$
$$dS_t = a(b - S)dt + \sigma SdW_t$$

Random Walk?
(E.g. GBM)

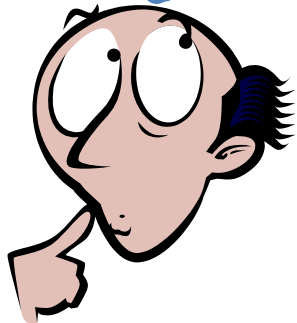
$$dS_t = \mu S_t dt + \sigma S_t dW_t$$



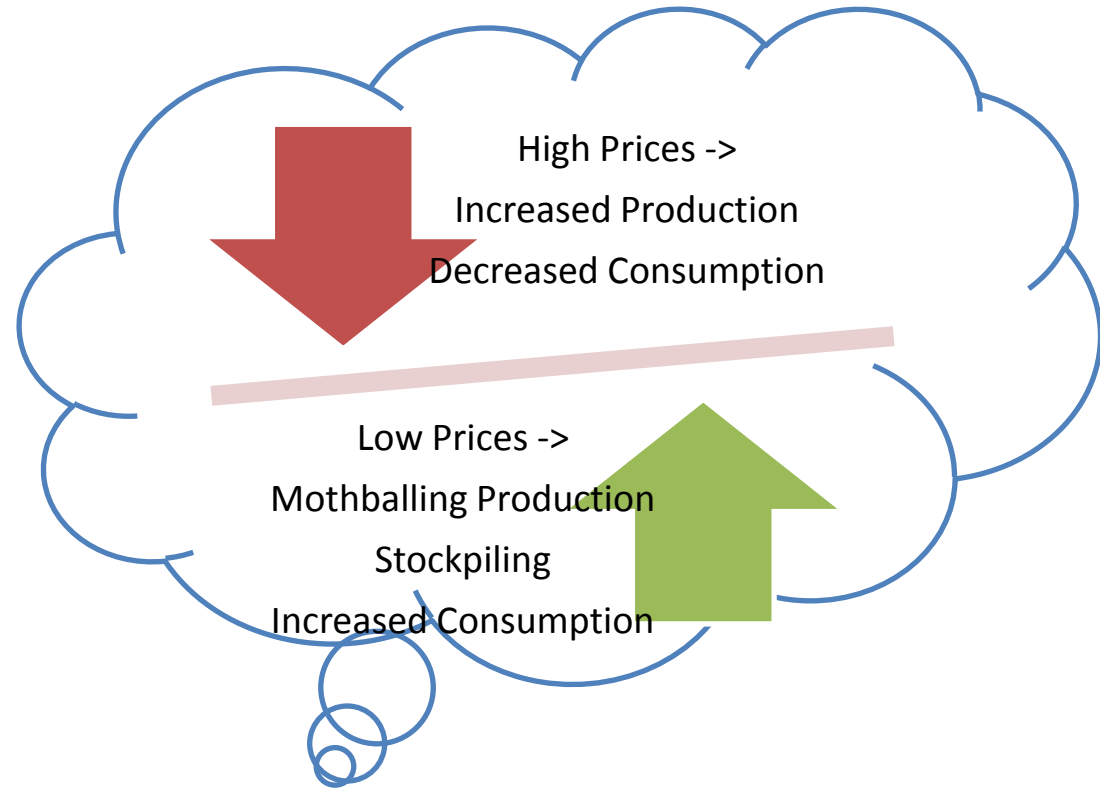
Crude Oil – Basic Spot Model

Mean Reverting
or Random Walk?

ADF Test,
P-P Test,
 $\gamma(0)$, $\gamma(1)$



Econometrics
guy



Economics
guy



Crude Oil – Basic Spot Model

Reduced
Form Models

$$dS_t = a(b - S)dt + \sigma dW_t$$
$$dS_t = a(b - S)dt + \sigma S dW_t$$
$$dS_t = \mu S_t dt + \sigma S_t dW_t$$



Finance /
Econometrics
guy

Structural
Models

Supply =
Demand =
Price = f(Supply,
Demand)



Economics
guy



Crude Oil - WTI

What is WTI anyway?

Quantity

- 1000 US barrels

Quality

- Specific domestic crudes with 0.42% sulfur by weight or less, not less than 37° API gravity nor more than 42° API gravity. The following domestic crude streams are deliverable: West Texas Intermediate, Low Sweet Mix, New Mexican Sweet, North Texas Sweet, Oklahoma Sweet, South Texas Sweet.
- Specific foreign crudes of not less than 34° API nor more than 42° API. The following foreign streams are deliverable: U.K. Brent, for which the seller shall receive a 30 cent per barrel discount below the final settlement price, Norwegian Osberg Blend is delivered at a 55¢-per-barrel discount, Nigerian Bonny Light, Qua Bore, and Colombian Cusiana are delivered at 15¢ premium.

Time

- on or after the first calendar day and completed by the last calendar day of the delivery month

Place

- Cushing, Oklahoma

From http://www.nymex.com/CL_spec.aspx



Crude Oil – Delivery Date

What is WTI anyway?

Time

- “on or after the first calendar day and completed by the last calendar day of the **delivery month**”
- February 2010 WTI – deliver in Feb 2010



Crude Oil – Delivery Date

Interest Rates

- Term Structure, observed now, of future **interest rates**

Yield Curve

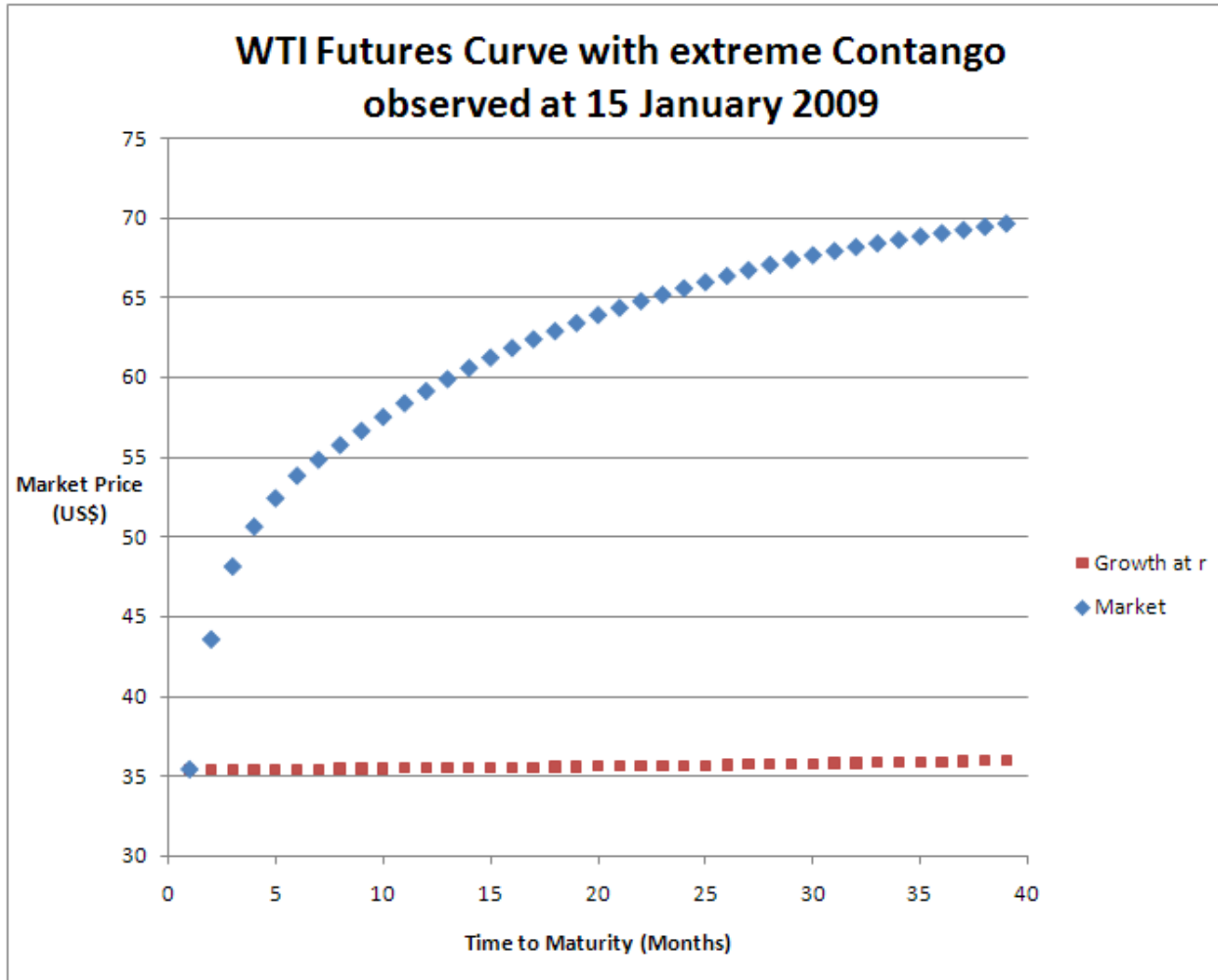
Commodities

- Term structure, observed now, of future **prices**

Futures Curve

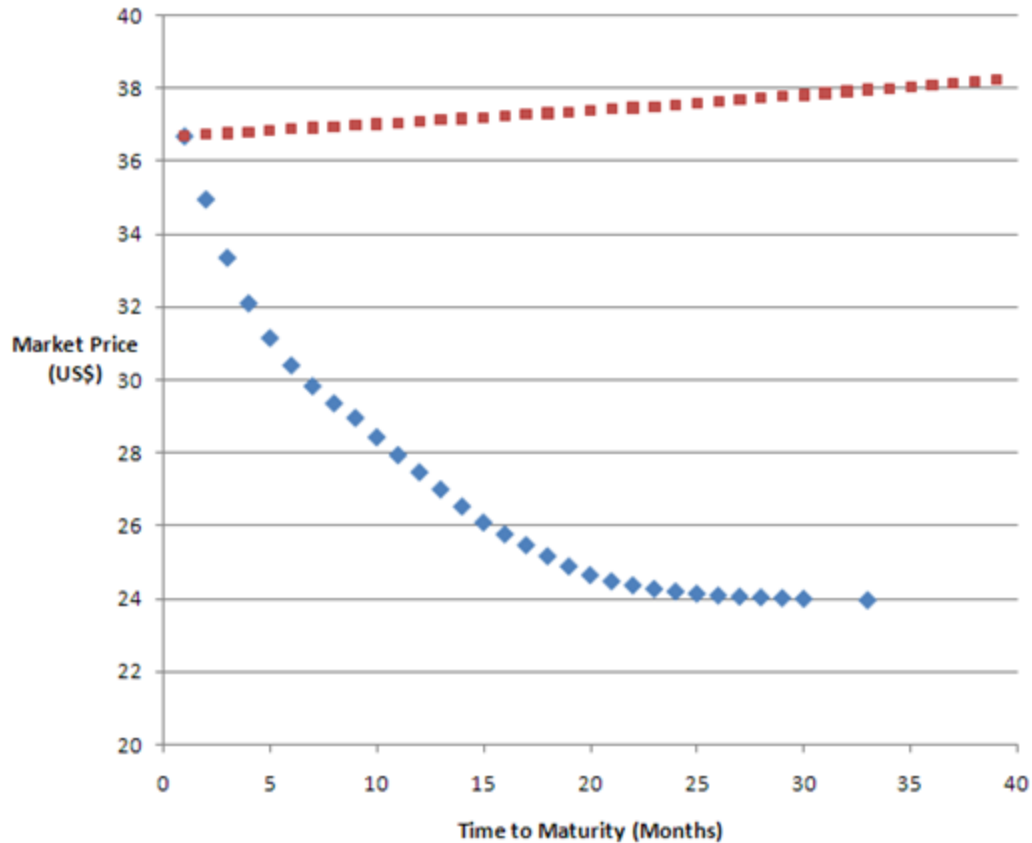


Crude Oil – Futures Curves





Crude Oil – Futures Curves





Futures Curves

Most commodity trading is in futures

Spot-forward relationship

$$F_{t,T} = S_t e^{(r-y)(T-t)}$$

There's an huge amount more to know about futures curves....

Natural Gas



Much more complicated –
makes oil seem simple!



Natural Gas

Regional

- Expensive to move
- 3 world markets

Seasonal

- Mainly for heating / cooling

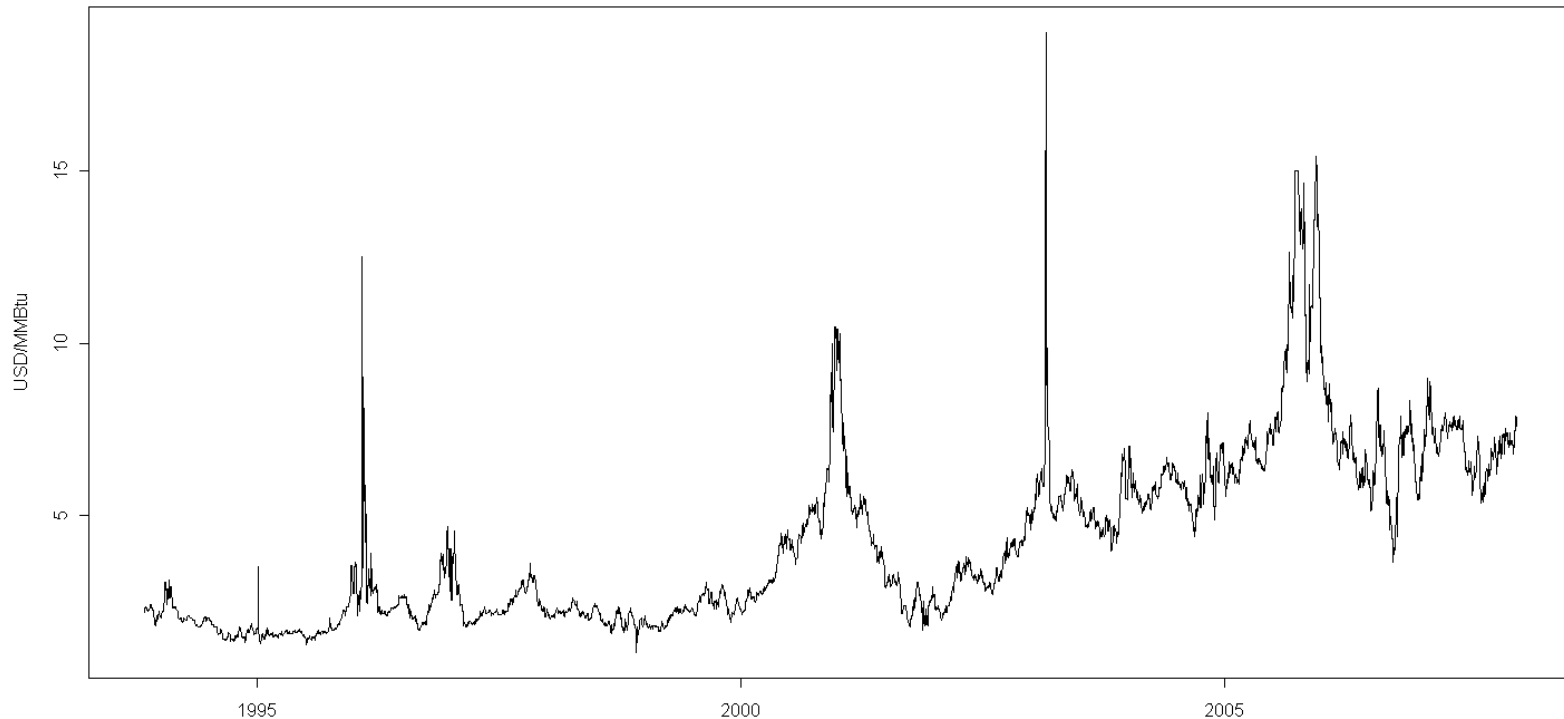
Political risk

- Usually only 1 route to market



Natural Gas – Spot Price

Henry Hub (USA) Natural Gas Spot Price, 1995-2007





Source : Steve Ohana MSc lecture notes,
<http://www.ems.bbk.ac.uk/faculty/ohana/>




Natural Gas – 3 Regions

Regionality – Model as 3 correlated processes?


$$dS_{1,t} = a(b - S_{1,t})dt + \sigma S_{1,t}dW_{1,t}$$


$$dS_{2,t} = a(b - S_{2,t})dt + \sigma S_{2,t}dW_{2,t}$$


$$dS_{3,t} = a(b - S_{3,t})dt + \sigma S_{3,t}dW_{3,t}$$

$$\text{cov}(dW_1, dW_2) = \rho_{1,2}$$

$$\text{cov}(dW_1, dW_3) = \rho_{1,3}$$

$$\text{cov}(dW_2, dW_3) = \rho_{2,3}$$



Natural Gas – 3 Regions

Regionality – Can it really be so easy?



$$dS_{1,t} = a(b - S_{1,t})dt + \sigma S_{1,t}dW_{1,t}$$

OR

$$dS_{1,t} = a_1(b_1 - S_{1,t})dt + \sigma_1 S_{1,t}dW_{1,t}$$

↑ ↑ ↑



Natural Gas – Spot Price

All's not well in SDE land...

$$dS_{1,t} = a_{1,t}(b_{1,t} - S_{1,t})dt + \sigma_{1,t} S_{1,t}dW_{1,t}$$

$$db_{1,t} = \mu_{b1} b_{1,t}dt + \theta_{b1} b_{1,t}dW_{b1,t}$$

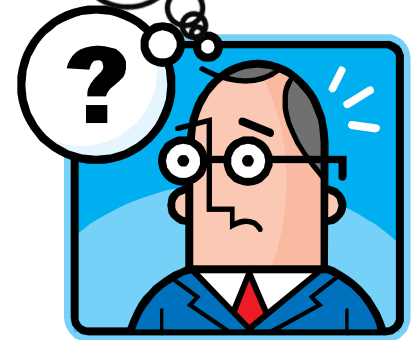
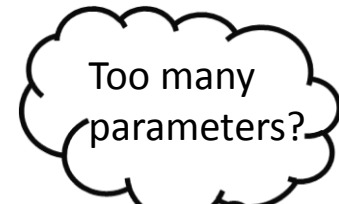
$$d\sigma_{1,t} = ???$$

$$\text{cov}(dW_1, dW_{b1}) = ???$$

Stochastic and region-specific volatility

Stochastic and region-specific mean

Stochastic and region-specific reversion speed





Natural Gas – Advanced Modelling

Back at the office, lots more jargon is heard....



Seasonality?

Jumps?
Spikes?

Markov Regime
Switching?

Fat Tails
Non Gaussian
Excess Kurtosis
Levy Processes

Volumetric Risk?
Disruption Risk?

Hubbert Peak?
Depletion Rate?

**Conclusion : fit the
model to the problem**



Electricity

Breaks the usual definition of “something physical”

Odd in many ways

But fits in the energy family nicely



Electricity - Basics

Un-storable

Hard to Hedge

Price Spikes

Expensive to Move

Regional Markets

Price Spikes

Demand Varying and Inelastic

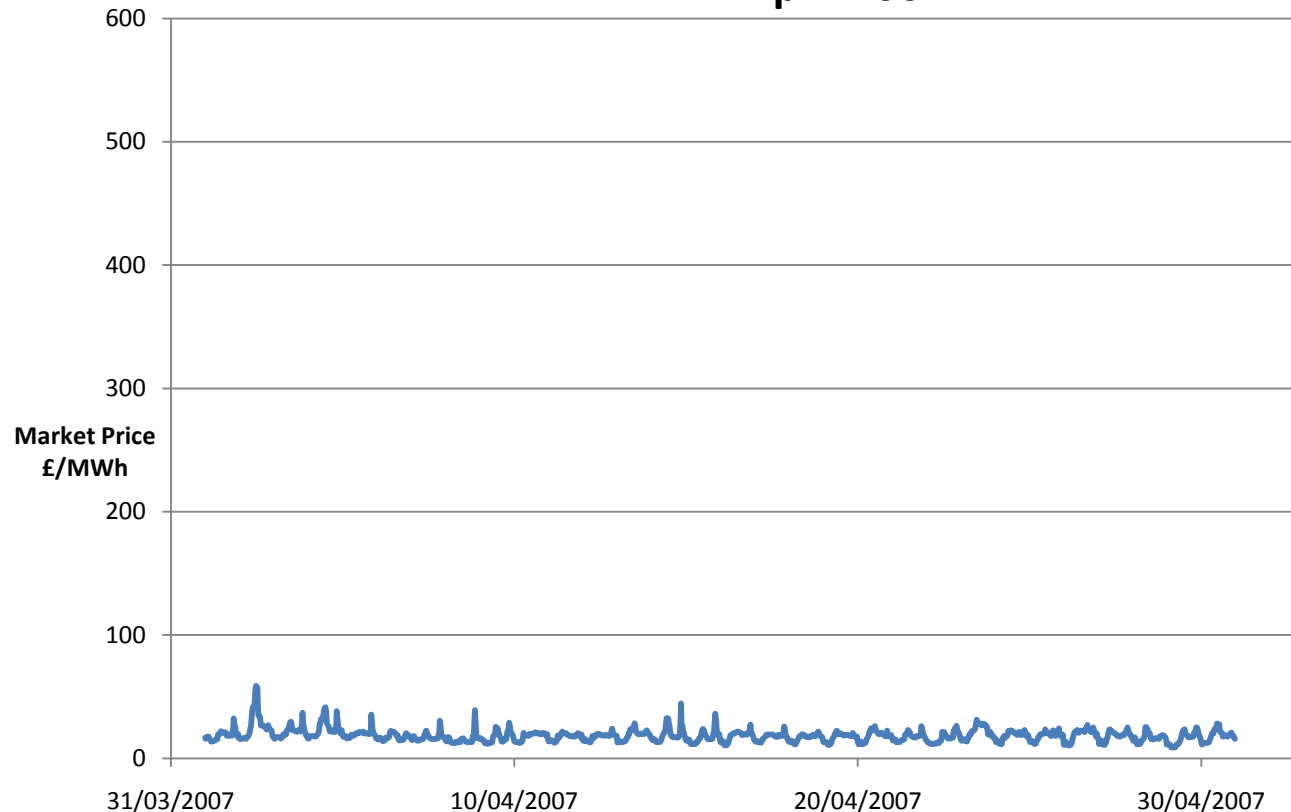
Supply HAS to Fit Demand

Price Spikes



Electricity – Summer

Price of wholesale electricity in Great Britain in the short-term market
April 2007



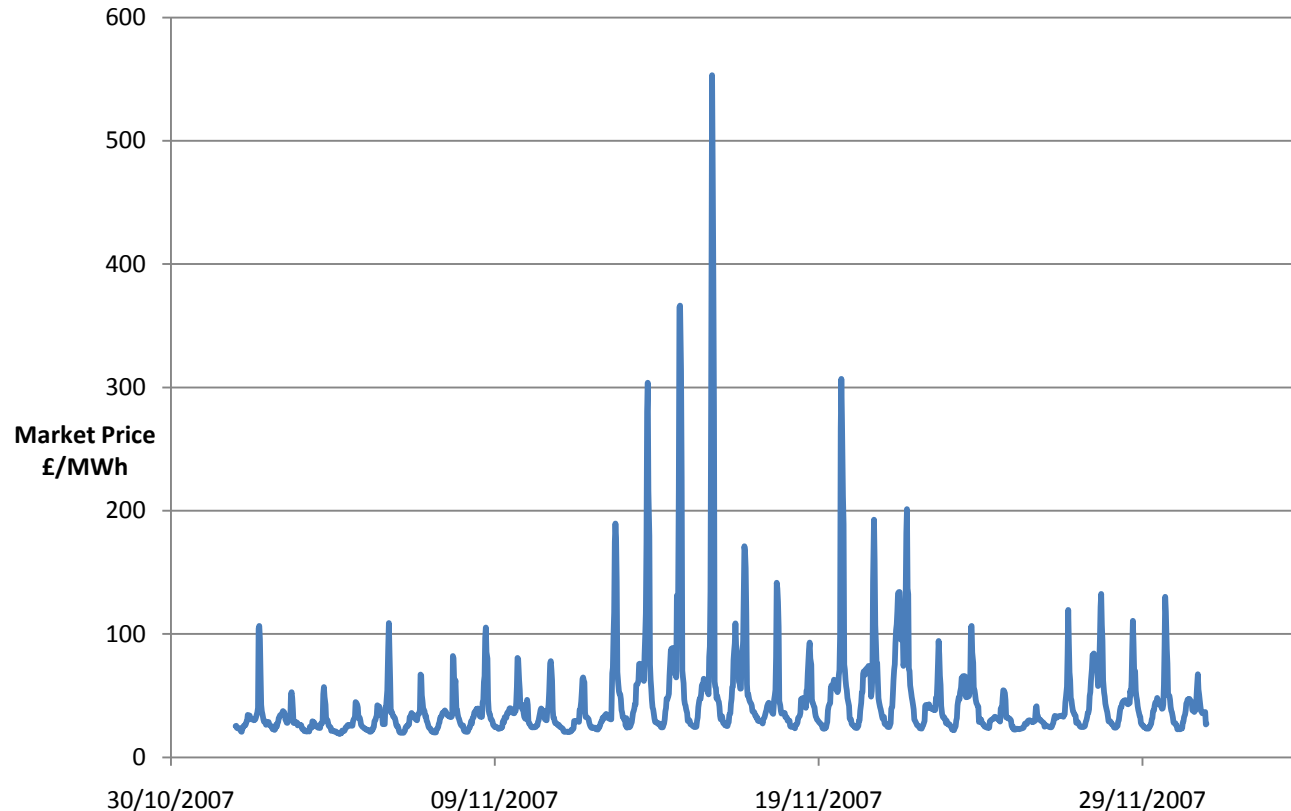
Source : <http://www.elexon.co.uk/marketdata/PricingData/MarketIndexData/default.aspx>
(they manage UK electricity market)



Electricity – Spikes!

Price of wholesale electricity in Great Britain in the short-term market

November 2007



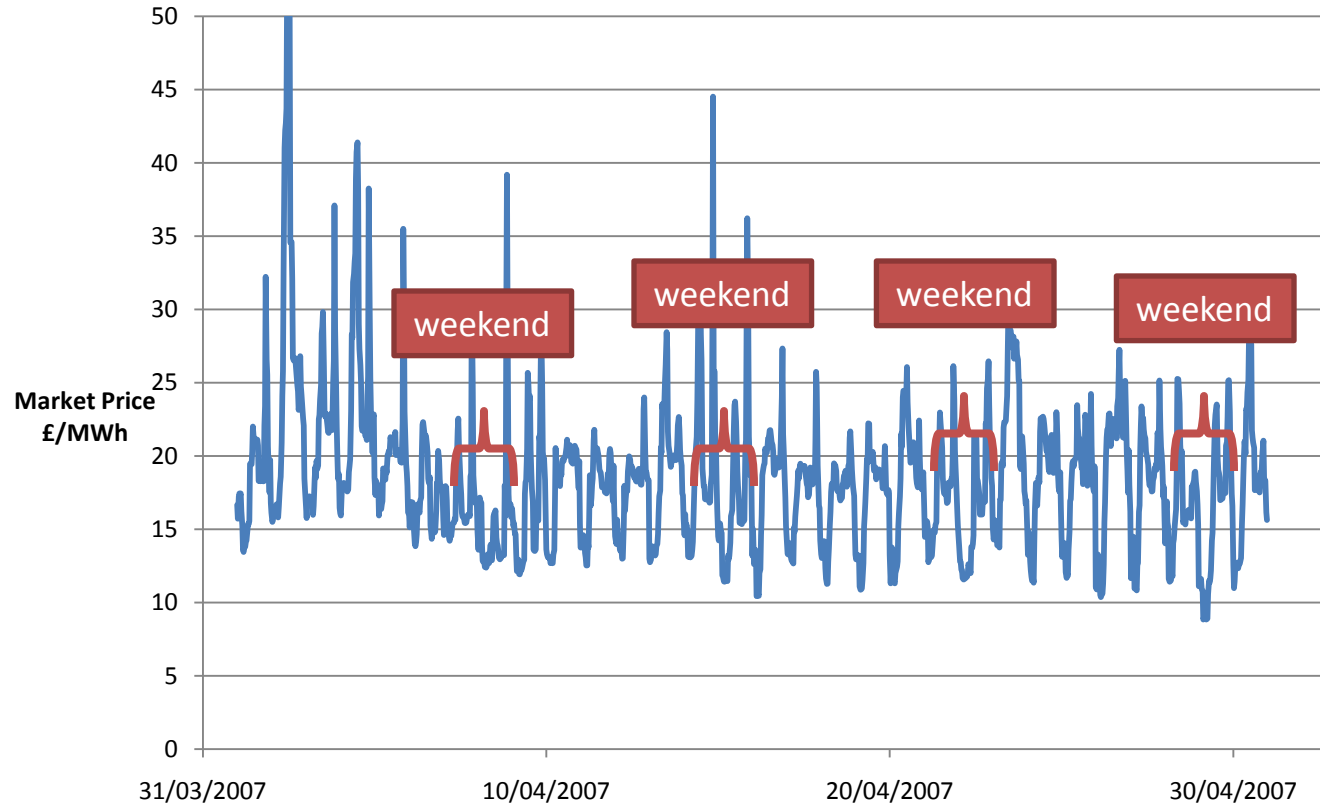
Source : <http://www.elexon.co.uk/marketdata/PricingData/MarketIndexData/default.aspx>

(they manage UK electricity market)



Electricity – Seasonality

Price of wholesale electricity in Great Britain in the short-term market
April 2007

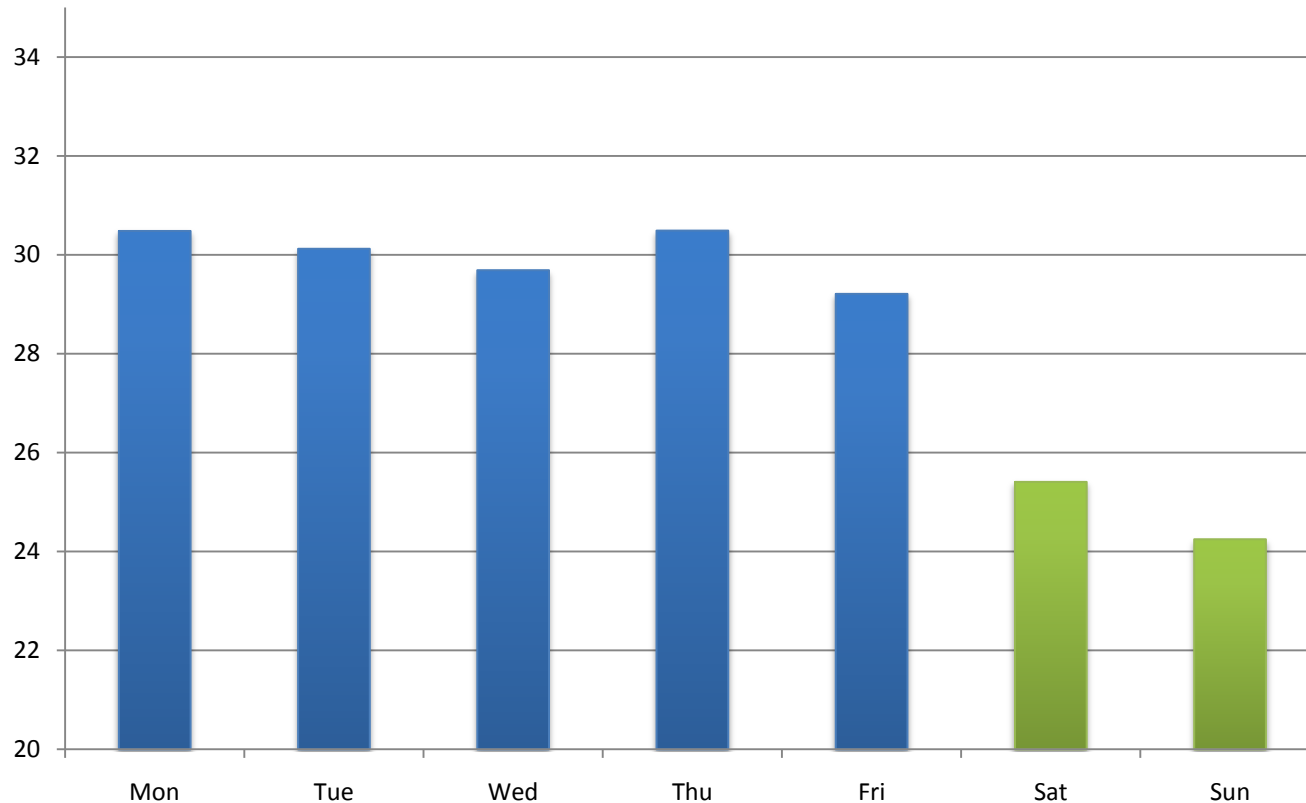


Source : <http://www.elexon.co.uk/marketdata/PricingData/MarketIndexData/default.aspx>
(they manage UK electricity market)



Electricity – Seasonality

Daily Average Electricity Price (£/MWh, 2007)

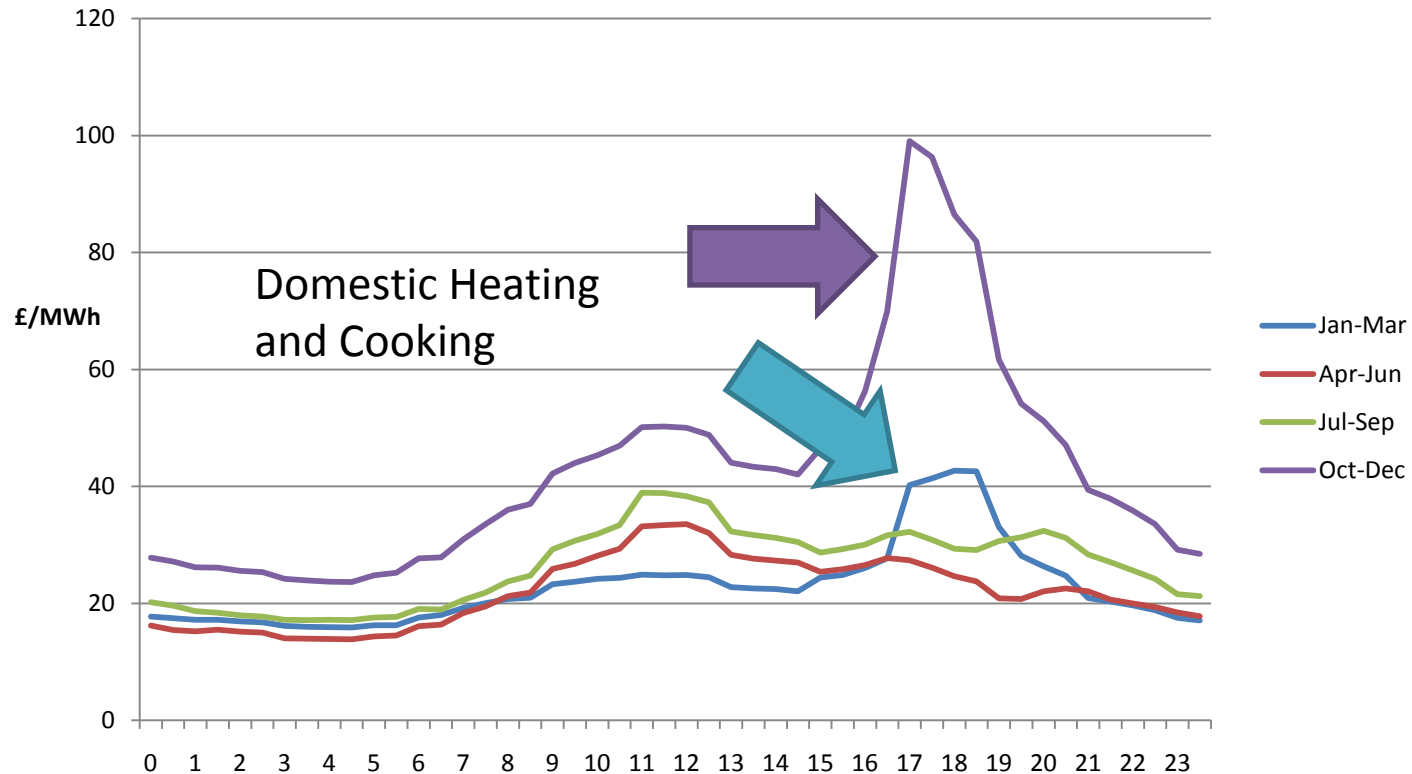


Source : <http://www.elexon.co.uk/marketdata/PricingData/MarketIndexData/default.aspx>
(they manage UK electricity market)



Electricity – Seasonality

UK Electricity 2007, Half-Hourly Prices over the Day



Source : <http://www.elexon.co.uk/marketdata/PricingData/MarketIndexData/default.aspx>
(they manage UK electricity market)

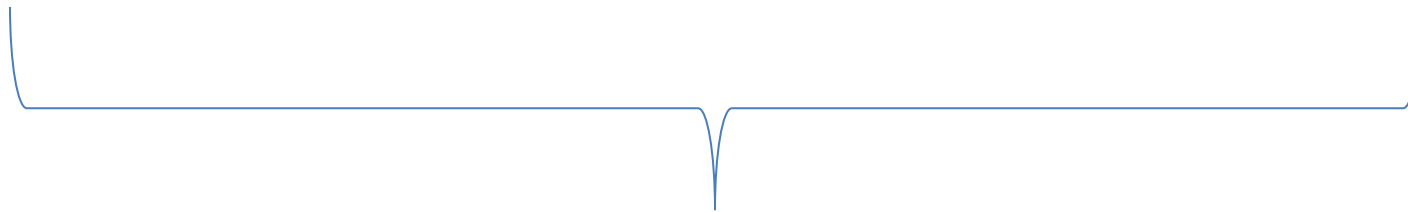


Temperature Data

WeatherData
Edinburgh
London

MeanTemperature 2008
MeanTemperature 2008

Daily
Daily



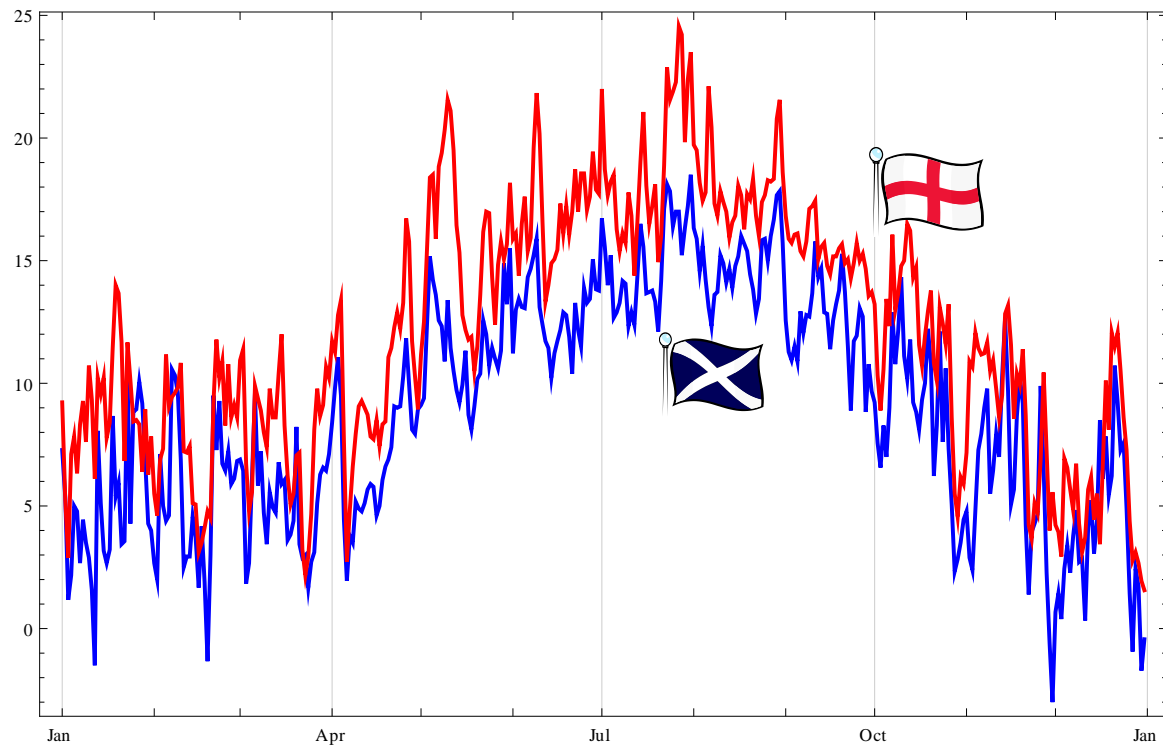
Say I wanted to know something like this, to investigate electricity price vs. temperature



Temperature Data

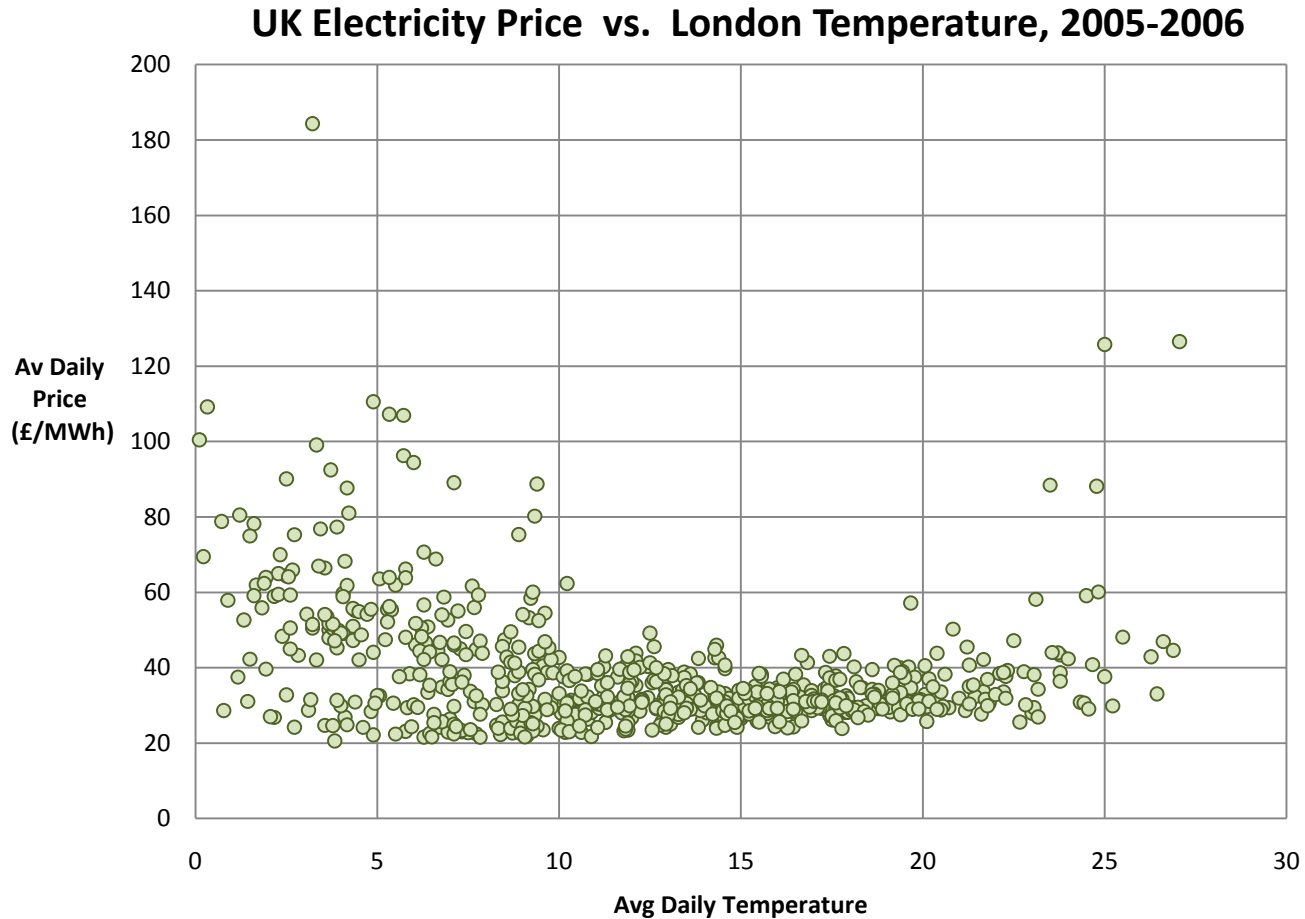
```
DateListPlot[  
{WeatherData[{"Edinburgh","UnitedKingdom"}, "MeanTemperature", {{2008}, {2008}, "Day"}],  
WeatherData[{"London","UnitedKingdom"}, "MeanTemperature", {{2008}, {2008}, "Day"}]  
},  
Joined->True, PlotStyle->{{Blue,Thick}, {Red,Thick}}
```

Mathematica is a great
source of weather
data! (version 7+)





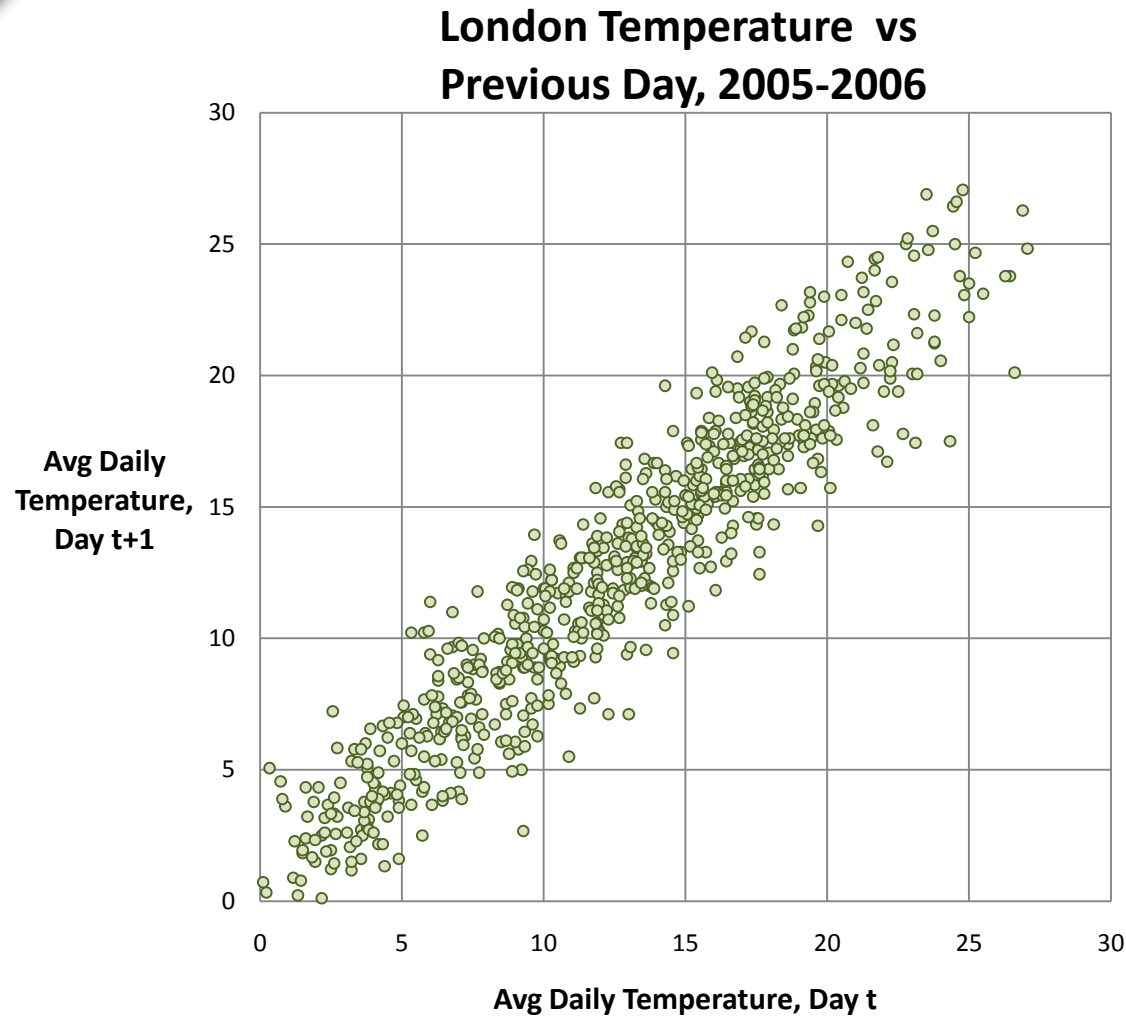
Electricity and Temperature



Source : <http://www.elexon.co.uk/marketdata/PricingData/MarketIndexData/default.aspx>
and Mathematica 7 data source



Temperature Autocorrelation



Source : Mathematica 7 data source



Jumps vs Spikes

Onyx Pharmaceuticals Inc. (ONXX)

1:10PM ET: 25.59 ↑0.44 (1.75%)

More On ONXX

Quotes

Summary
Real-Time
Options
Historical Prices

Charts

Interactive
Basic Chart
Basic Tech. Analysis

News & Info

Headlines
Financial Blogs
Company Events
Message Board

Company

Profile
Key Statistics
SEC Filings
Competitors
Industry
Components

Analyst Coverage

Analyst Opinion
Analyst Estimates
Research Reports
Star Analysts



- See your CQF notes on jump-diffusion
- First developed by Merton in 1975

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t + J_t dQ_t$$

$$dQ_t \sim P(\lambda)$$

$$dW_t \sim N(0, \sqrt{dt})$$

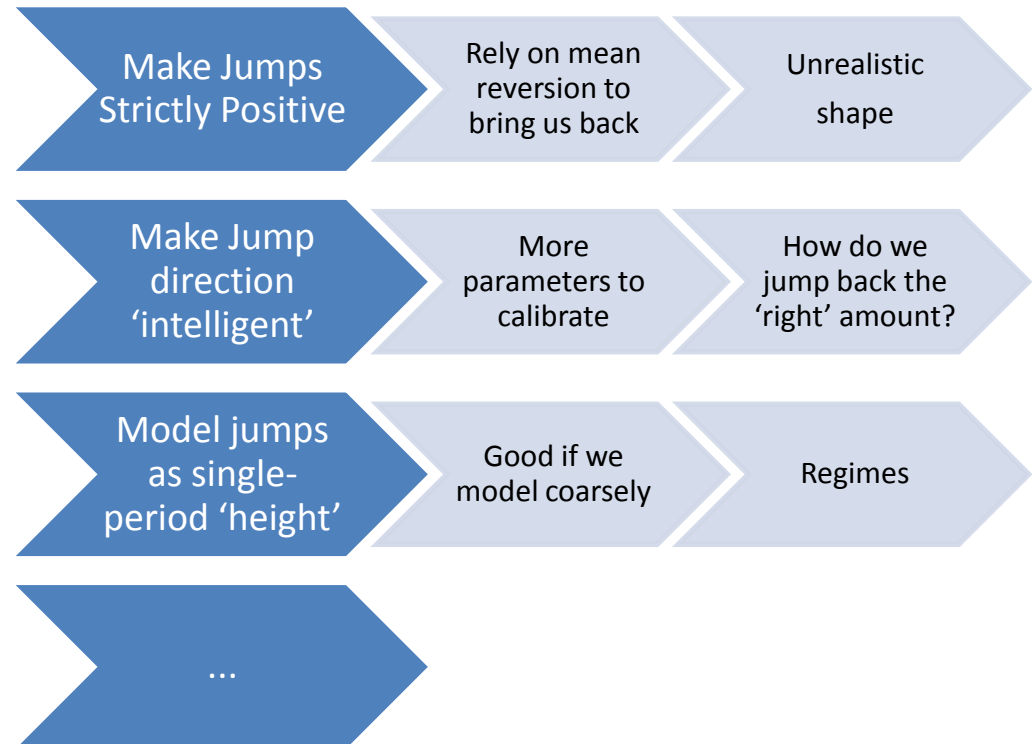
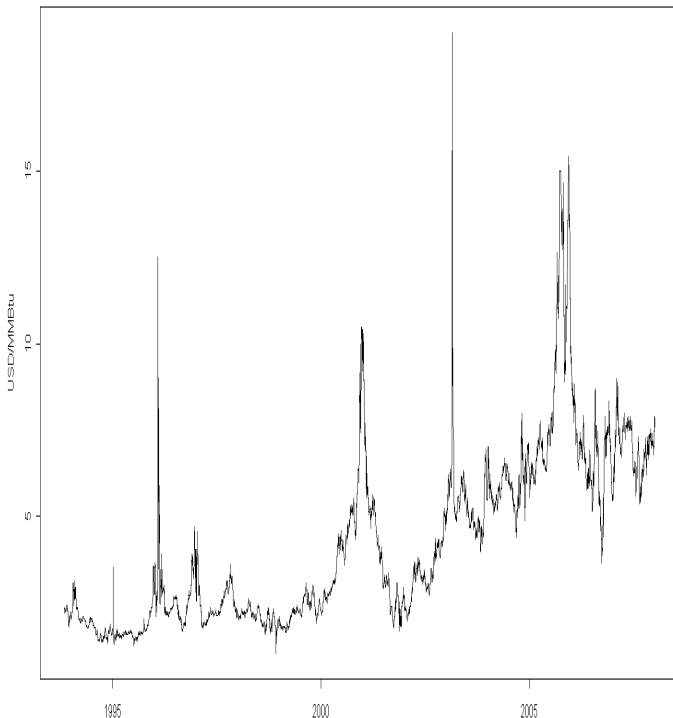
$$J_t \sim N(0, \beta)$$

Graph Source : Yahoo Finance



Jumps vs Spikes

Henry Hub (USA) Natural Gas Spot Price, 1995-2007



Data Source : Steve Ohana MSc lecture notes,
<http://www.ems.bbk.ac.uk/faculty/ohana/>



Markov Regime Switching

Given ...

... then the probability of ...

$$\begin{array}{l} \textit{current state} \\ \textit{next state} \end{array} \begin{bmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{bmatrix}$$

E.g.

State 1 = Brownian Motion, low volatility

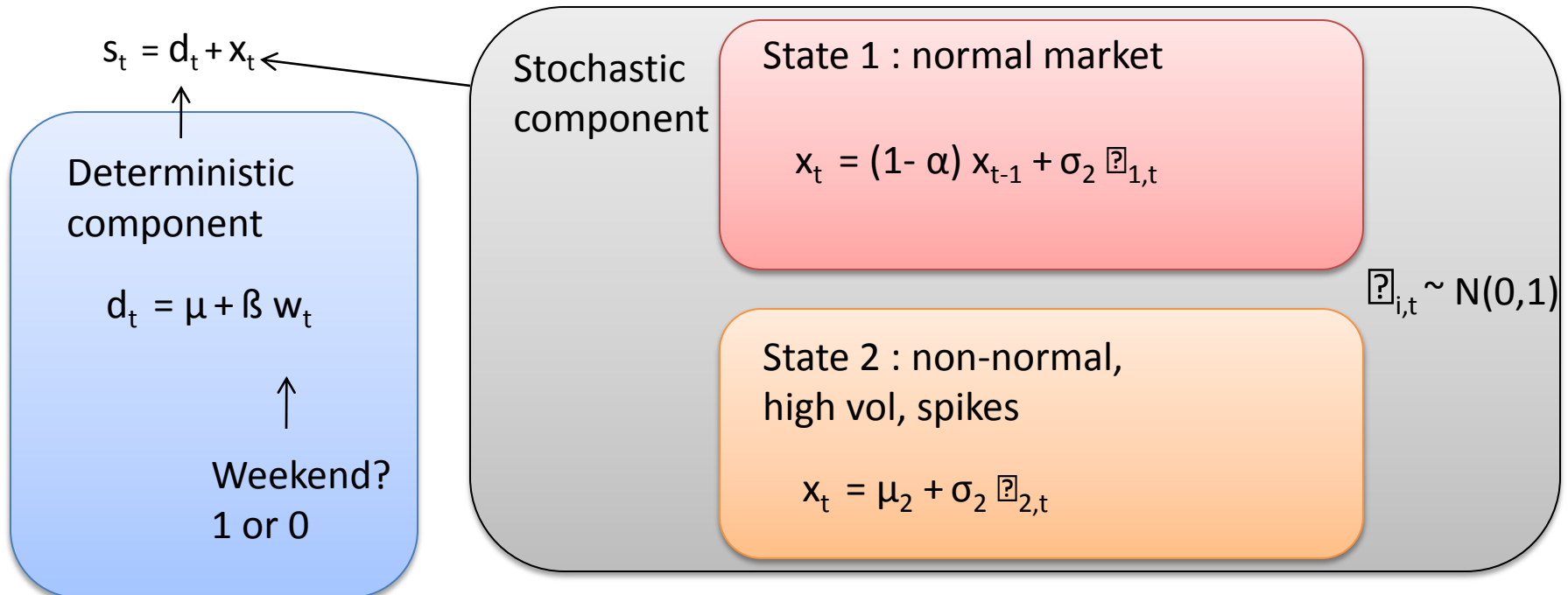
State 2 = Jump Regime / high volatility



Let's Model Electricity

“The influence of temperature on spike probability in day-ahead power prices”
Ronald Huisman, Erasmus University Rotterdam, Energy Economics, 2008

Let s_t be the natural logarithm of the day-ahead price for delivery of 1 MWh in day t





Let's Model Electricity (Matlab, VBA is similar)

```
function [ ] = Model1( start_price, start_regime, start_date, end_date )  
%Calculate next day's price based on model1 of  
% "The influence of temperature on spike probability in day-ahead power  
% prices" by Ronald Huisman, 2007.  
% http://papers.ssrn.com/sol3/papers.cfm?abstract_id=992369
```

```
mu = 3.993;  
mu2 = 0.260;  
beta1 = -0.480;  
  
p = [ 0.957  0.043  ; ...  
      0.201  0.799];
```

```
alpha = 0.157;  
sigma1 = 0.227;  
sigma2 = 0.635;
```

```
s = zeros(1,end_date-start_date+1);  
x = zeros(1,end_date-start_date+1);  
d = zeros(1,end_date-start_date+1);  
price = zeros(1,end_date-start_date+1);  
regime = zeros(1,end_date-start_date+1);
```

```
% Initial conditions.  
x(1) = log(start_price);  
s(1) = log(start_price);  
price(1) = start_price;  
regime(1) = start_regime;
```

```
for t=2:1:end_date-start_date+1;  
    date = start_date + t - 1;
```

```
% next regime transition...  
regime(t) = regime(t-1); % unless we switch, stay the same.  
if (regime(t-1) == 1 && rand(1) < p(1,2))  
    regime(t) = 2;  
elseif (regime(t-1) == 2 && rand(1) < p(2,1))  
    regime(t) = 1;  
end
```

```
% w = weekend dummy variable  
[ n, ignoreme ] = weekday(date);  
w = 0;  
if n == 1 || n == 7  
    w = 1;  
end
```

```
% deterministic part  
d(t) = mu + beta1 * w;
```

```
% stochastic part  
if (regime(t) == 1)  
    x(t) = (1-alpha)*x(t-1) + sigma1*randn(1);  
else  
    x(t) = mu2 + sigma2*randn(1);  
end
```

```
% s = d+x  
s(t) = d(t) + x(t);
```

```
price(t) = exp(s(t));  
display(sprintf('Day %s (weekend %d) price %f', datestr(date), w, price(t)));  
end
```

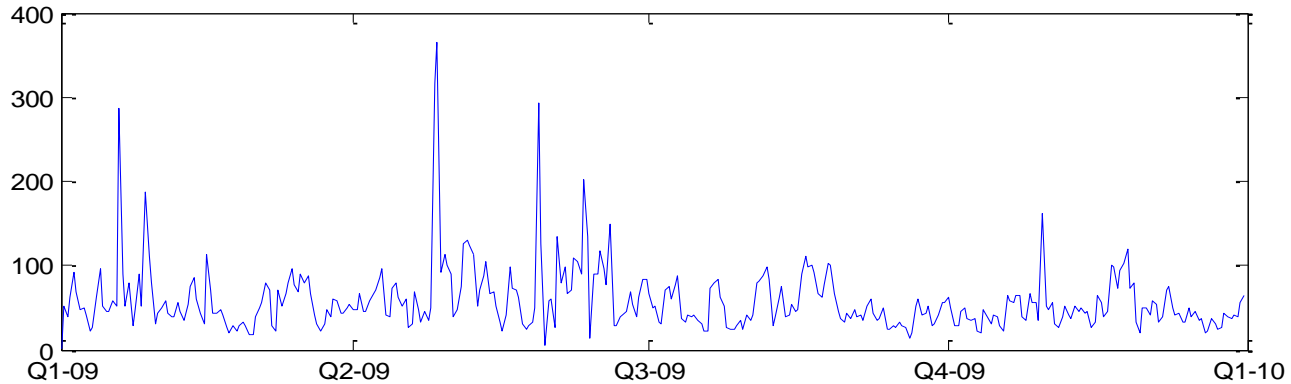
```
subplot (2,1,1);  
plot([start_date:1:end_date],price);  
datetick('x');  
subplot (2,1,2);  
plot([start_date:1:end_date], regime);  
datetick('x');
```

```
end
```

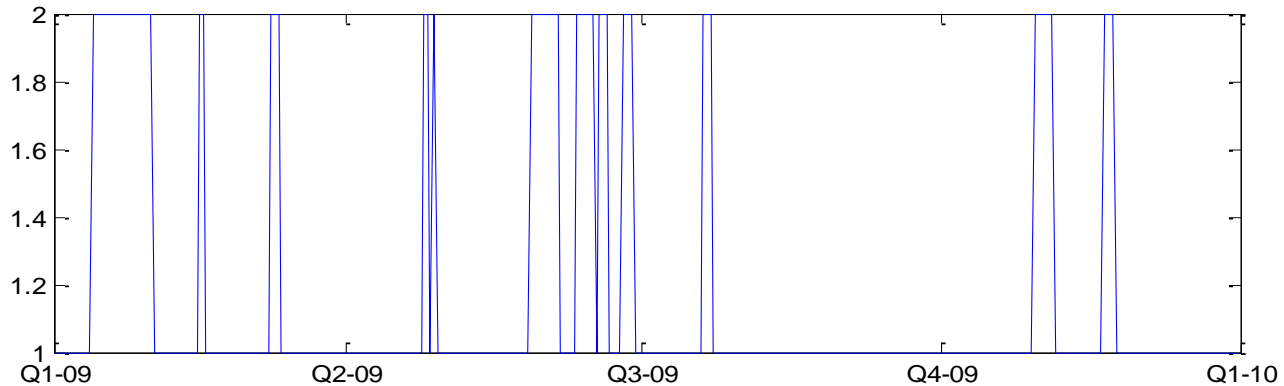


Let's Model Electricity

Price



Regime





Let's Model Electricity

Huisman's Paper - Further Models

Model 2 – Mean Varies with Temperature

- Model distance of temperature from mean temperature.
- $d_t = \mu + \beta w_t + \beta | \text{distance of temperature from average} |$

Model 3 – Greater Regime Switching during Extreme Temperatures

- Parameters in transition matrix change by linear factor related to $| \text{distance of temperature from average} |$

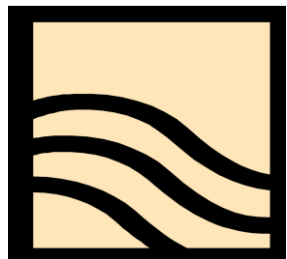
Model 4 – Greater Regime Switching during Extreme Temperatures and unexpected temperatures

- Captures not only summer and winter periods, but days that are hotter (summer) or colder (winter) than the recent days (maybe power stations aren't prepared for the sudden increase in demand)



Summary of Energy Modelling

Different Product – Different Model
Non-Normal
Seasonal
Weather Dependent

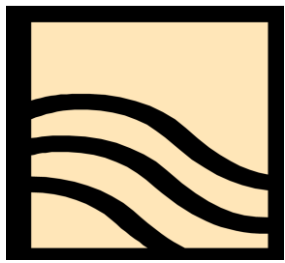


Commodity Correlation and Cointegration

More linkages that you might expect

Across commodities

Across space and time



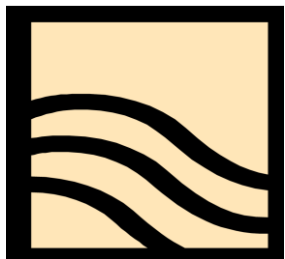
Commodity Correlation

Formally: Correlation & Co-integration

- Short run (innovations), vs long-run (levels)

Reasons for co-integration

- I will cover several on the next pages

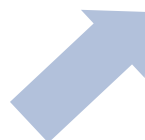


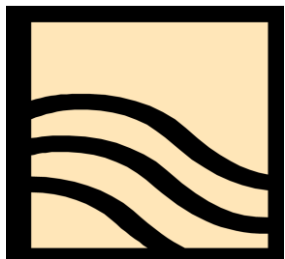
Commodity Correlation

Substitution : A or B



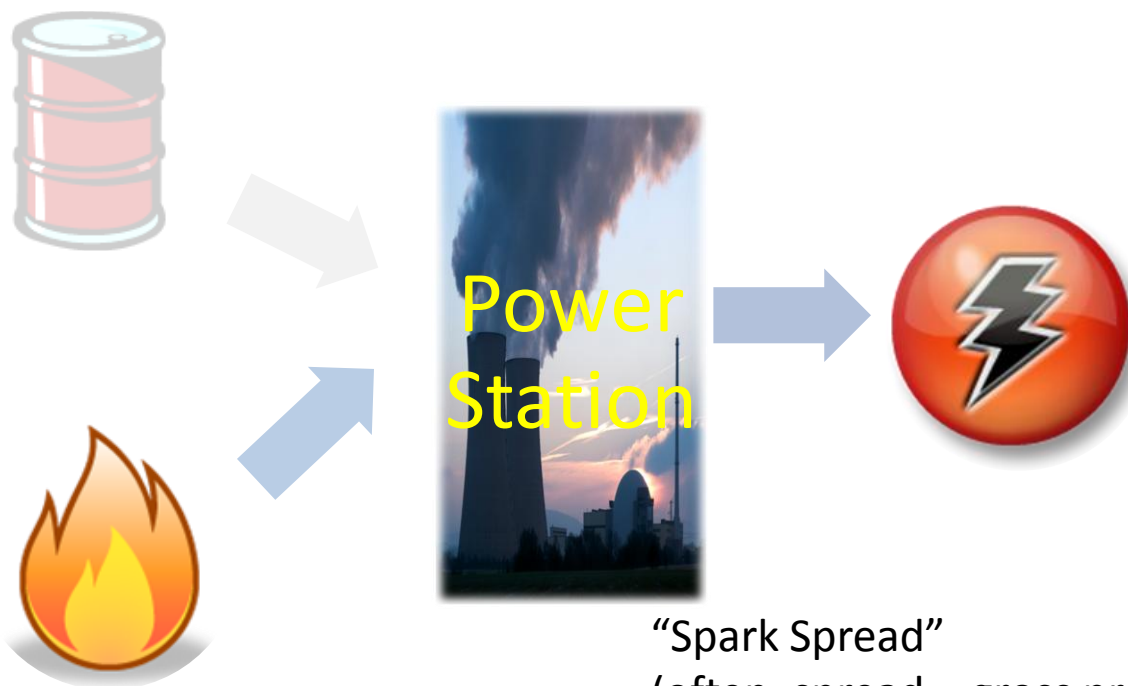
OR



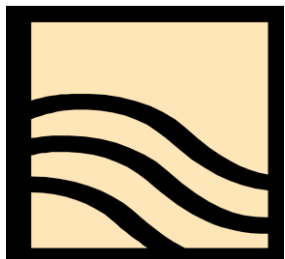


Commodity Correlation

Input: A needed for B



“Spark Spread”
(often, spread = gross profit)
Spreads are often traded as derivatives



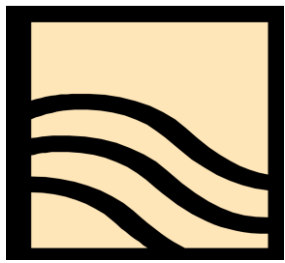
Commodity Correlation

Alternative Output : A can produce B or C



OR



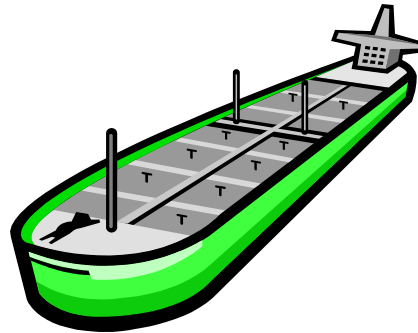


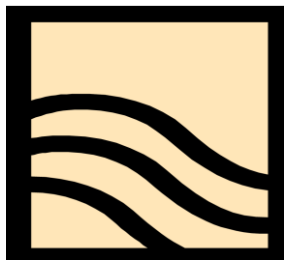
Commodity Correlation

Resource Competition



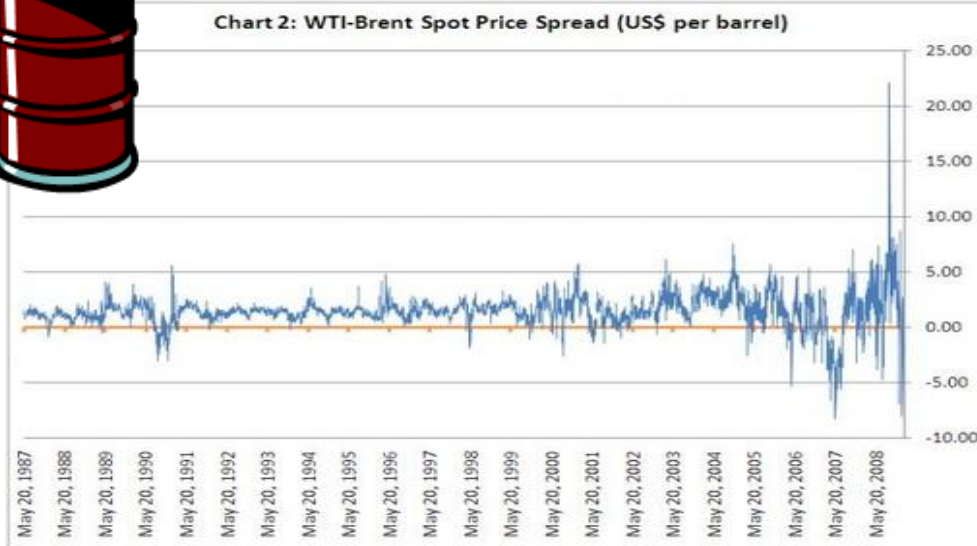
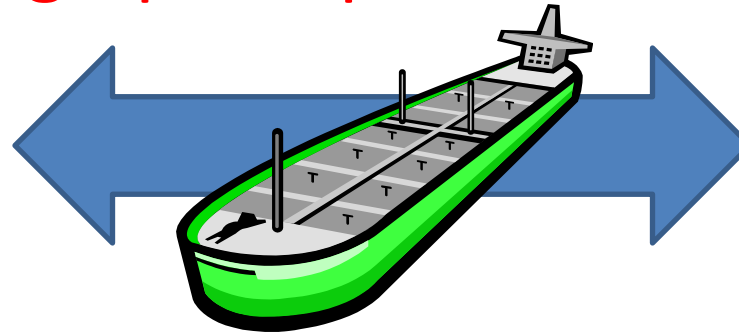
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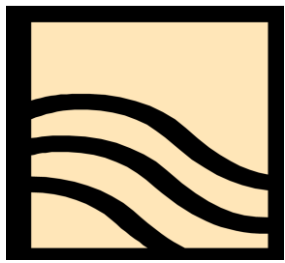


Correlation - Spreads

Geographic Spreads - Move

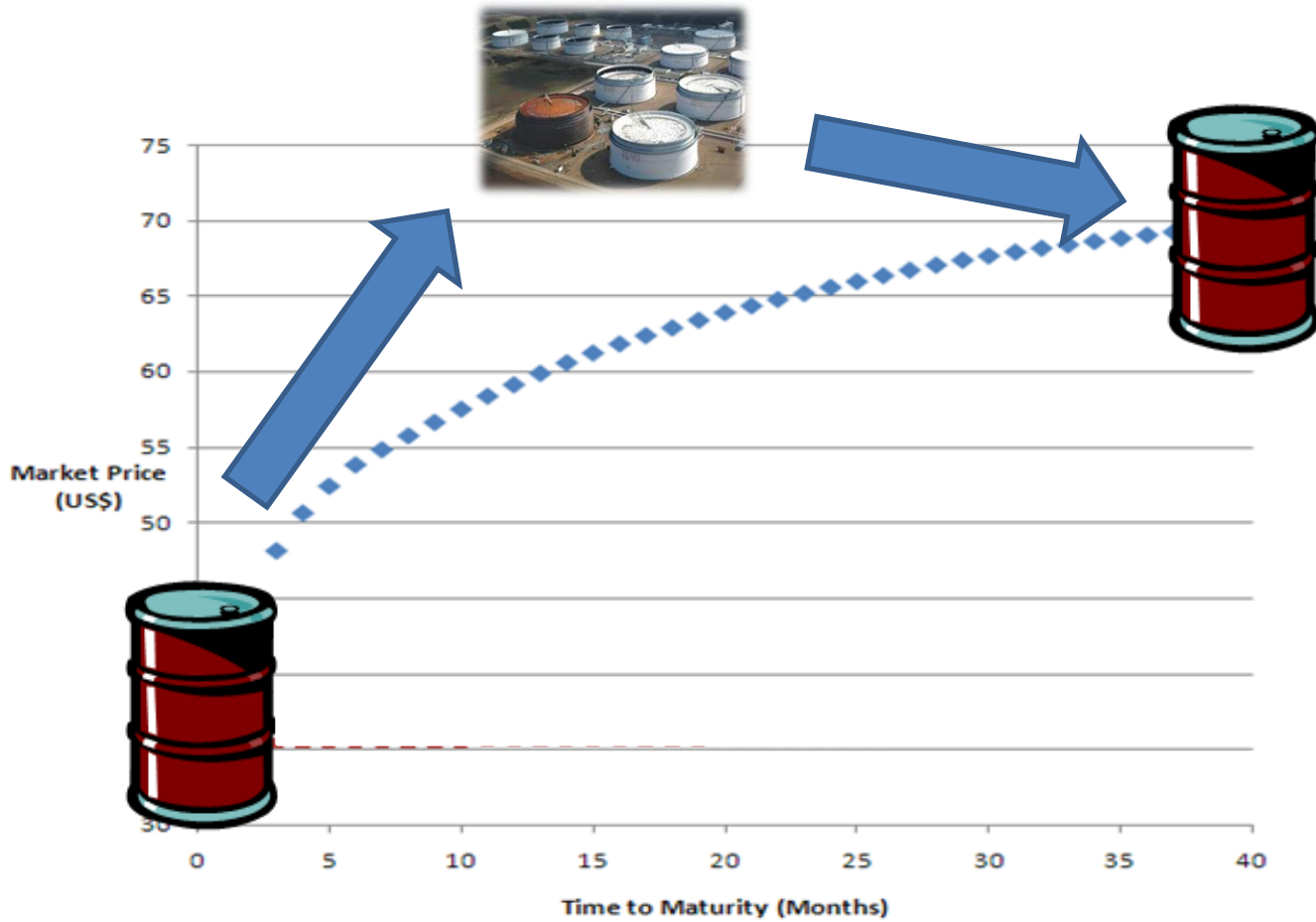


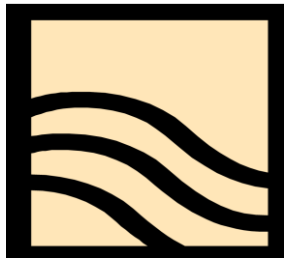
Graph Source : www.rgemonitor.com



Correlation - Spreads

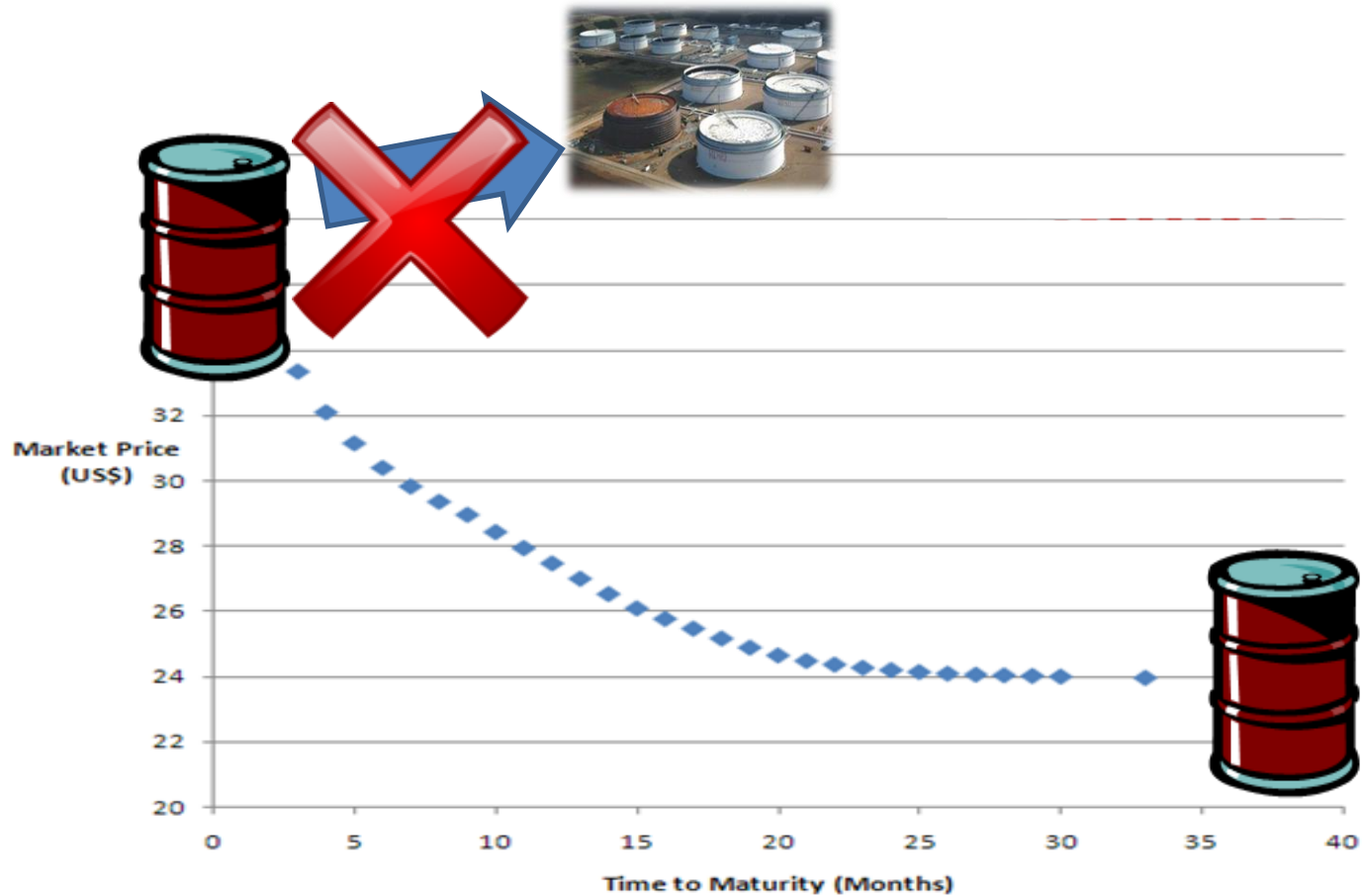
Temporal Spreads - Store

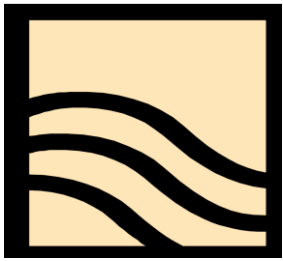




Correlation - Spreads

Temporal Spreads - Store





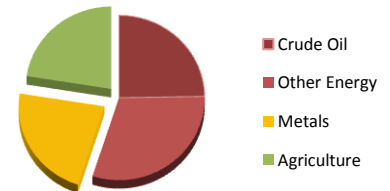
Correlation – Other Reasons

Dollar Effect

- Dollar  , Commodities 

Fund Effect

- “Give me \$100 of commodity exposure please”



Summary



What Are Commodities?



Energy Modelling



Commodity Correlation

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