Key Papers, Models and Studies Related to Commodity Modelling

A Brief Summary of the Most Important Papers and Models

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Introduction

I have compiled below, a list of some of the most important and influential papers and models, of interest to those studying commodities finance or involved in modelling commodities (oil, gas, electricity, metals and agricultural products). It is not exhaustive; it is intended as a guide to 'where to get started'. References in each paper should be used as a guide to further study. Within each section, papers are listed chronologically, I thank my PhD supervisor Hélyette Geman for this insight.

The below papers are not necessarily the easiest papers to read, they are just the most influential. For a list of easy-to-read papers that introduce various topics in commodities, see my article 'Recommended Papers' at www.CommodityModels.com/recommended-papers .

Key Finance Papers of Direct Relevance to Commodities

Author & Date	Title	Key Concepts and Importance	Full Reference	URL (only official listed, but try also Google Scholar)
Bachelier, 1900	The Theory of Speculation *(in French).*	Introduction of *arithmetic* Brownian motion as a model of asset prices, and a demonstration of how this can be used to price options. Unfortunately, this paper was relatively unknown to the finance world until around 1965. It's rediscovery ultimately led to the Black & Scholes model, described below.	Bachelier, L. The Theory of Speculation. *Annales scientifiques de l'École Normale Supérieure* Sér. 3, no. 17: 21-86.	http://www.numdam.org/item?id=ASENS_1900_3_17__21_0
Black & Scholes, 1973	The Pricing of Options and Corporate Liabilities	Pricing model using the *geometric* Brownian motion and incorporating dynamic delta hedging. Closed form solutions for the price of 'vanilla' put and call options provided some assumptions are made.	Black, Fischer, and Myron Scholes. 1973. The Pricing of Options and Corporate Liabilities. *The Journal of Political Economy* 81, no. 3 (June): 637-654.	http://www.jstor.org/stable/1831029
Merton, 1975	Option pricing when underlying stock returns are discontinuous	Extends the Black & Scholes 1973 paper to include the possibility of 'jumps' in the price process of the underlying security, which he models using a 'Poisson' process.	Merton, Robert. 1975. Option pricing when underlying stock returns are discontinuous.	http://dspace.mit.edu/handle/1721.1/1899.
Black, 1976	The pricing of commodity contracts	Extends Black & Scholes 1973 paper to include options on futures, particularly important in the commodities world.	Black, Fischer. 1976. The pricing of commodity contracts. *Journal of Financial Economics* 3, no. 1-2 (January): 167-179.	http://dx.doi.org/10.1016/0304-405X(76)90024-6
Vasicek, 1977	An equilibrium characterization of the term structure	Early model for interest rates. Uses the mean reverting 'Ornstein-Uhlenbeck' process as a model for the short-term interest rate, later widely used in modelling any mean-reverting process. The weakness of this model is that it can generate negative interest rates.	Vasicek, Oldrich. 1977. An equilibrium characterization of the term structure. *Journal of Financial Economics* 5, no. 2 (November): 177-188..	http://dx.doi.org/10.1016/0304-405X(77)90016-2

Margrabe, 1978	The Value of an Option to Exchange One Asset for Another	If we have two assets both following geometric Brownian motions (as in Black&Scholes), Margrabe derives a formula for the value of the option to convert one into the other (i.e. sell one, and receive the other). This is very relevant to commodities because (for example) a power station can be considered as an option (you don't have to run it every day) on the ability to convert gas (or oil, etc) into electricity. A risk of using the model is that it assumes g.b.m. for both assets, constant volatility, and crucially, constant correlation between the two asset returns.	Margrabe, William. 1978. The Value of an Option to Exchange One Asset for Another. *The Journal of Finance* 33, no. 1 (March): 177-186.	http://www.jstor.org/stable/2326358
Cox, Ingersoll, Ross, 1985	A Theory of the Term Structure of Interest Rates	Most important feature is that it introduces a mean-reverting process that cannot go below 0, by introduction of a 'square root' term.	Cox, John C., Jonathan E. Ingersoll, and Stephen A. Ross. 1985. A Theory of the Term Structure of Interest Rates. Econometrica 53, no. 2 (March): 385-407.	http://www.jstor.org/stable/1911242
Heston, 1993	A closed-form solution for options with stochastic volatility with applications to bond and currency options	Classic model with a stochastic volatility. Stock prices follows a geometric Brownian motion, and variance follows a mean-reverting model of the square-root (CIR) type.	Heston, S. L. 1993. A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Review of financial studies*: 327–343.	http://www.jstor.org/stable/2962057

Commodities Specific Papers

Author & Date	Title	Key Concepts and Importance	Full Reference	URL (only official listed, but try also Google Scholar)
Hoteling, 1931	The economics of exhaustible resources	One of the key results in a fairly complex paper is that, net of extraction costs, the prices of an exhaustible commodity such as oil should rise at the risk-free rate 'r'.	Hotelling, H. 1931. The economics of exhaustible resources. *The Journal of Political Economy* 39: 137–175.	?
Working, 1949	The Theory of Price of Storage	The first few pages simply describe what we now call a forward curve (a name not yet used at this time). Explains why shape of the forward curve is related to the cost of storing the commodity.	Working, H. 1949. The theory of price of storage. *The American Economic Review* 39: 1254–1262.	http://www.jstor.org/stable/1816601
Hubbert, 1956	Nuclear Energy and the Fossil Fuels	Prediction of shape of growth curve of oil and gas production, both US and worldwide, and predictions for dates when production would peak.	Hubbert, M King. 1956. Nuclear Energy and the Fossil Fuels. *Drilling and Production Practise, American Petroleum Institute* (June).	http://www.hubbertpeak.com/Hubbert/1956/1956.pdf
Samuelson, 1965	Proof that properly anticipated prices fluctuate randomly	The most widely cited result of this paper is that the volatility of distant futures of an asset (those with longer maturity) is lower than the volatility of nearby futures, which he describes as being 'a well-known rule of thumb'. This has become known as the 'Samuelson effect'. Don't confuse with a different paper, which he also published in 1965, entitled 'Proof that properly *discounted present values* of assets *vibrate* randomly'.	Samuelson, P.A. 1965. Proof that properly anticipated prices fluctuate randomly. *Industrial Management Review* 6: 41-49.	http://www.ifa.com/Media/Images/PDF%20files/Samuelson-Proof.pdf
Brennan & Schwartz, 1985	Evaluating Natural Resource Investments	Probably the first paper to apply the theory of 'real options' to commodities. They consider a mine, producing a resource whose price can be modelled as a geometric Brownian motion (g.b.m). They then determine the optimal behaviour: to run the mine (and at what rate to produce the output), or to close the mine ready to re-open later, or even to abandon it.	Brennan, Michael J, and Eduardo S Schwartz. 1985. Evaluating Natural Resource Investments. *Journal of Business* 58, no. 2: 135-57.	http://www.jstor.org/stable/2352967

Fama & French, 1987	Commodity futures prices: Some evidence on forecast power, premiums, and the theory of storage	Discussion and analysis of what causes the 'convenience yield' observed in commodity futures curves. They provide a test to determine whether it is caused by the 'stockout' theory or the 'risk premium' theory.	Fama, E. F., and K. R. French. 1987. Commodity futures prices: Some evidence on forecast power, premiums, and the theory of storage. *Journal of Business* 60, no. 1 (January): 55-73.	http://www.jstor.org/stable/2352947
Gibson & Schwartz, 1990	Stochastic convenience yield and the pricing of oil contingent claims	Classic early commodities pricing model. The spot price is modelled as a geometric Brownian motion, and the convenience yield as a mean-reverting Ornstein-Uhlenbeck process.	Gibson, R., and E. S. Schwartz. 1990. Stochastic convenience yield and the pricing of oil contingent claims. *Journal of Finance* 45, no. 3: 959-976.	http://www.jstor.org/stable/2328801
Deaton & Laroque, 1992	On the behaviour of commodity prices	They examine the effects of storage on commodity prices, using a yearly model. They show that empirical prices are auto-correlated (at a yearly level), and show positive skew, i.e. the up-tail is more extreme than the down-tail. Assuming i.i.d. yearly harvests does not give this result unless we add storage. When we add storage, we see that very low prices become less likely, since producers will store the product. Also, high prices are unlikely in times of poor harvest, because we can extract from storage. However, occasionally the storage is empty and we will observe price spikes. This model therefore helps to explain the empirical data.	Deaton, Angus, and Guy Laroque. 1992. On the Behaviour of Commodity Prices. *The Review of Economic Studies* 59, no. 1 (January): 1-23.	http://www.jstor.org/stable/2297923
Litzenberger & Rabinowitz, 1995	Backwardation in oil futures markets: Theory and empirical evidence	Show that 'backwardation' is the most common shape for the crude oil futures curve (the so called 'normal backwardation' hypothesis, nothing to do with the normal distribution). Classical theory says this must be because oil extraction costs are expected to fall. They propose an alternative reason : producers are able to defer their production, giving an embedded optionality and increasing the price of short-term contracts.	Litzenberger, R. H, and N. Rabinowitz. 1995. Backwardation in oil futures markets: Theory and empirical evidence. *Journal of Finance*: 1517–1545.	http://www.jstor.org/stable/2329325

Bessembinder & Lemmon, 2002	Equilibrium pricing and optimal hedging in electricity forward markets	Looking specifically at electricity markets, they show that forward prices contain a component that is related to risk premium.	Bessembinder, H., and M. L Lemmon. 2002. Equilibrium pricing and optimal hedging in electricity forward markets. *Journal of Finance* 57, no. 3: 1347–1382.	http://www.jstor.org/stable/2697781
Borovkova & Geman, 2006	Seasonal and stochastic effects in commodity forward curves	Provides a method of splitting the prices of seasonal commodities into seasonal and non-seasonal components, via their forward curves. In some markets, such as electricity, the 'spot' price is hard to observe and therefore model. This paper proposes a method of constructing an alternative state variable, as a proxy for the spot price, using futures prices.	Borovkova, Svetlana, and Hélyette Geman. 2006. Seasonal and stochastic effects in commodity forward curves. Review of Derivatives Research 9, no. 2: 167-186. doi:10.1007/s11147-007-9008-4.	http://dx.doi.org/10.1007/s11147-007-9008-4

Essential Econometrics Techniques

Author & Date	Title	Key Concepts and Importance	Full Reference	URL (only official listed, but try also Google Scholar)
Granger, 1969	Investigating Causal Relations by Econometric Models and Cross-spectral Methods	Introduces the concept of 'Granger-causality', i.e. that a change in a time-series at time *t* might cause a corresponding change in a different time-series at time *t+1*, and that we can test this using a standard regression technique.	Granger, C. W. J. 1969. Investigating Causal Relations by Econometric Models and Cross-spectral Methods. *Econometrica* 37, no. 3 (August): 424-438.	http://www.jstor.org/stable/1912791
Dickey & Fuller, 1979	Distribution of the Estimators for Autoregressive Time Series With a Unit Root	Introduces the 'Dickey-Fuller' test (later extended to the Augmented Dickey-Fuller or ADF test) to determine whether a time series is stationary (mean-reverting) or has a unit-root (non-mean-reverting).	Dickey, David A., and Wayne A. Fuller. 1979. Distribution of the Estimators for Autoregressive Time Series With a Unit Root. *Journal of the American Statistical Association* 74, no. 366 (June): 427-431.	http://www.jstor.org/stable/2286348